Worksheet Monday 3/31 Solutions

Exercises:

1. Find all the critical points of $f(x) = x^3 - x^2$ and decide if they are local maxima, minima or neither.

$f'(x) = 3x^2 - 2x = x(3x-2)$

Crit. points $x = 0,\ x = 2/3$.

$f''(x) = 6x - 2$

$f''(0) = -2 < 0$ local max

$f''(2/3) = 2 > 0$ local min

$x = 0$ max
$x = 2/3$ min

2. Repeat Problem 1 for the function $g(x) = (x-5)^3 + 2$.

$g'(x) = 3(x-5)^2$

Crit. points
$x = 5$

$g''(x) = 6(x-5)$

$g''(5) = 0$ neither

$x = 5$, Neither

3. For which values of a and b does,

$$f(x) = axe^{bx^2}$$

obtain a global maximum value of 1 at $x = 2$?

$f'(x) = ae^{bx^2} + ax(2bx)e^{bx^2} = (a + 2abx^2)e^{bx^2}$

$f'(2) = 0 \iff a(1 + 2b(4)) = 0 \iff a \neq 0,\ b = -1/8$

$f(2) = a(2)(e^{-1/2}) = 1 \Rightarrow a = \frac{1}{2}e^{1/2}$.

$a = \frac{1}{2}e^{1/2},\ b = -1/8$

4. Find the global extrema of the following:

(a) $x - 2\arctan(x)$ on the interval $[0, 4]$

(b) $x^2 \ln(x)$ on $[0.5, 4]$

(c) $x + 1/x$ on $[0.2, 4]$

(d) $2\cos(x) + 2\sin(x)$ on $[0, \pi/2]$

A.) $f'(x) = 1 - \frac{2}{1+x^2} = \frac{x^2-1}{x^2+1}$

Crit. point $x = 1$
Check $x = 0, 4$

x	$f(x)$	
0	0	neither
1	$1 - \pi/2$	global min
4	1.34	global max.

B.) $(4, 16\ln(4))$ max
$(1/2, \frac{1}{4}\ln(\frac{1}{2}))$ min

C.) $(1, 2)$ min
$(0.2, 5.2)$ max

D.) min. $(0, 2)$ & $(\pi/2, 2)$
max $(\pi/4, 2\sqrt{2})$

5. Why can't $f(x) = x^{101} + x^{51} + x + 1$ have any local extrema?

$f'(x) = 101x^{100} + 51x^{50} + 1$ has no zeros ($x^{100}, x^{50} \geq 0$

$\Rightarrow 101x^{100} + 51x^{50} + 1 > 0$)

6. Draw a graph or write a formula for a function which satisfies each of the following conditions.

(a) A continuous function on $[0, 1]$ with no critical points.

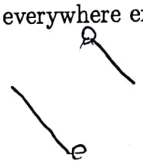
$f(x) = x$

(b) A continuous function on $[0, 1]$ with neither its global maximum nor minimum occurring at the endpoints.

(c) A continuous function on $[0, 1]$ that obtains its global maximum four times: twice at the endpoints and twice in the interior.

(d) A continuous function on $(0, 1)$ that has a local maximum and minimum but obtains neither a global minimum nor a global maximum.

(e) A function on $[0, 1]$ which is continuous everywhere except $1/2$ and has neither a global maximum nor a global minimum.



(f) A continuous function on $(0, 1)$ that has a critical point but no local or global extrema.

(g) A continuous function on $(0, 1]$ that has no global maxima or minima.

$\frac{1}{x}\sin\left(\frac{1}{x}\right)$

(h) A continuous function on $\mathbb{R}$ with infinitely many critical points but no local or global extrema.

Some step function